**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *m* = 45 minutes and *s* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: (A). 0.2659

Explanation:

The work begin after 10mins so the average time increase from 45min to 55min for normal distribution is Z=(X-µ)/

=(60-55)/8

=0.625

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *m* = 38 and Standard deviation *s* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: The current age (in years) of 400 clerical employees at an insurance claims processsing center is normally distributed with mean 38&SD=6

To find the True or False for A and B

Mean =38

SD=6

Z score=(value-mean)/SD

Z score for 44=(44-38)/6=1=84.13

People above 44 age =100-84.13=15.8763 out of400

Z score for 38=(38-38)/6=0=50%

Hence People between 38 & 44 age =84.13-50=34.13% 137 out of 400

Hence more employees at the processinng center are older than 44 than between 38 and 44 is FALSE

Z score for 30=(30-30)/6=-1.33=9.15% 36 out of 400

Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees is TRUE

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: As we know that if XN(, and yN(are two independent random variables then X+Y N, and X-Y Nsimilarly if Z=aX+bY , where X and Y are as defined above , I.e Z is linear combination of X and Y then ZN  
therefore in the equation

X1+x2NN

2=N

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: since we need to find out the values of a and b which are symmetric about the mean such that the probability of random variable taking a value between them is 0.99, we have to work out in inverse order

The probability of getting value between a and b should be 0.99

So the probability of going wrong or the probablility outside the a and b area is 0.01(ie.1-0.99)

The probability towards left from a= -0.005(ie.0.001/2)

The probability towards right from b=+0.005(ie.0.01/2)

So since we have the probabilities of a and b , we need to calculate X, the random variable at a and b which has got these probabilities

By finding the standard normal variable Z(Z value), we can calculate the X values

Z=(X-)/

For probability 0.005 the Z value is –2.57(from Z table)

Z\*=X

Z(-0.005)\*20+100=-(-2.57)\*20+100=151.4

Z(+0.005)\*20+100(-2.57)\*20+100=48.6

So, option D is the correct answer

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: given that $1=45

profit1N(5,)

profit2N(7,)

Thus ,

Company's profit

PN(5+7,)=N(12,)

A):95% of the probability lies between 1..96 standard deviation of the mean thus range is :

=(12-1.965,12+1.965)

=($2.2M,$22.8M)

=(Rs.99M,Rs1026M)

B):fifth percentile is calculated as :

P(Z P-12/5)=0.05

From P values of Z score table ,we get

P-12/5=-1.664

P=12-8.22=3.78

Thus at $3.78M dollars or rs 170.1M amount 5th percentile of profit lies or 5th percentile of profit is 170.1Million

C):Loss is when profit 0

Thus P<0

The first division of company thus have larger probability of making a loss in a given year